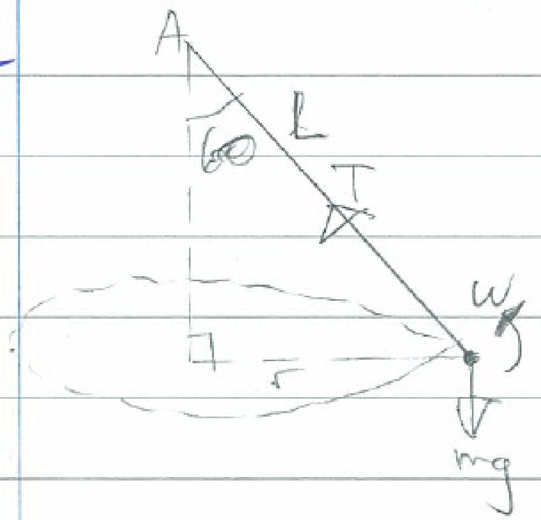
M3 - January 2004

1.



a) $\uparrow \quad mg = T\cos 60$

$$2mg = T$$

b) $\leftarrow \quad \boxed{F = ma}$

$$T\sin 60 = m\omega^2 r$$

$$2mg \times \frac{\sqrt{3}}{2} = m\omega^2 L \frac{\sqrt{3}}{2}$$

$$2g = \omega^2 L$$

$$\omega^2 = \frac{2g}{L}$$

$$\omega = \sqrt{\frac{2g}{L}}$$

c) $T = \frac{\lambda x}{a}$

$$2mg = \frac{\lambda\left(L - \frac{3}{5}L\right)}{\frac{3}{5}L}$$

$$2mg = \left(\frac{2}{5} \times \frac{5}{3}\right)\lambda$$

$$\lambda = 3mg$$

2 - a)

$$a = -4e^{-2t}$$

$$\frac{dv}{dt} = -4e^{-2t}$$

$$\int_1^v dv = -4\int_0^t e^{-2t}\,dt$$

$$[v]_1^v = \frac{-4}{-2}\left[e^{-2t}\right]_0^t$$

$$v - 1 = 2e^{-2t} - 2$$

$$v = 2e^{-2t} - 1$$

b) $v = 2e^{-2t} - 1$

$$0 = 2e^{-2t} - 1$$

$$\frac{1}{2} = e^{-2t}$$

$$-2t = \ln\frac{1}{2}$$

$$t = \frac{1}{2}\ln 2$$

$$\frac{dx}{dt} = 2e^{-2t} - 1$$

$$\int_0^x dx = \int_0^{\ln\sqrt{2}} (2e^{-2t} - 1)\,dt$$

$$[x]_0^x = \left[-e^{-2t} - t\right]_0^{\ln\sqrt{2}}$$

$$x = -\frac{1}{2} - \ln\sqrt{2} + 1$$

$$= \left(\frac{1}{2} - \ln\sqrt{2}\right)\text{m}$$

3. a) $F \propto \frac{1}{x^2}$ $\qquad F = \frac{k}{x^2}$

At the surface;

$\boxed{F = ma}$

$\frac{k}{R^2} = mg$

$k = mgR^2$

$\therefore F = \frac{mgR^2}{x^2}$

b) $\frac{1}{2} \times m \times u^2 = \frac{1}{2} \times m \times v^2 + \int_R^{2R} F \, dx$

$\frac{3}{4} mgR = \frac{1}{2} mv^2 + \int_R^{2R} \frac{mgR^2}{x^2} \, dx$

$3\cancel{m}gR = 2\cancel{m}v^2 + 4\cancel{m}gR^2 \int_R^{2R} \frac{1}{x^2} \, dx$

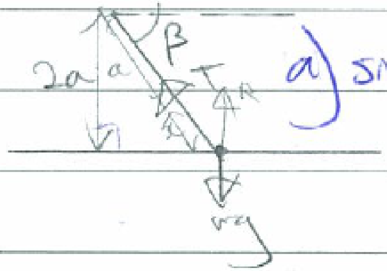
$$3gR = 2v^2 + 4gR^2\left[-x^{-1}\right]_R^{3R}$$

$$3gR = 2v^2 - \frac{4gR^2}{3R} + \frac{4gR^2}{R}$$

$$v^2 = \frac{1}{2}\left(3gR + \frac{4}{3}gR - 4gR\right) = \frac{1}{3}gR \times \frac{1}{2}$$

$$v = \sqrt{\frac{1}{6}gR}$$

4.



a) $\sin\beta = \frac{3}{5} = \frac{2a}{a+x}$

$$10a = 3a + 3x$$

$$7a = 3x$$

$$x = \frac{7a}{3}$$

$$E = \frac{\lambda x^2}{2a}$$

$$= \frac{\frac{1}{2}mg\cdot\left(\frac{7a}{3}\right)^2}{2a}$$

$$= \frac{49mga^2}{36a}$$

$$= \frac{49}{36}mga$$

b) $\frac{49}{36} mga = \frac{1}{2} mv^2 + \frac{mg \cdot a^2}{4a}$

$\frac{49}{18} ga - \frac{1}{2} ga = v^2$

$v = \sqrt{\frac{20}{9} ga} = \frac{2}{3}\sqrt{5ga}$

c) $T\sin\theta + R = mg$ (1)

$\sin\theta = \frac{2a}{a+x}$

$T = \frac{\lambda x}{a} = \frac{mgx}{2a}$ (2)

$a\sin\theta + x\sin\theta = 2a$

$x = \frac{2a - a\sin\theta}{\sin\theta}$

$= \frac{a(2-\sin\theta)}{\sin\theta}$ (3)

(3) in (2):

$T = \frac{mg}{2a} \cdot \frac{a(2-\sin\theta)}{\sin\theta}$

$= \frac{2mg - mg\sin\theta}{2\sin\theta}$ (4)

(4) in (1):

$\frac{2mg - mg\sin\theta}{2\sin\theta}, \sin\theta \neq R - mg$

$mg(2-\sin\theta) + 2R = 2mg$

$2R = 2mg - mg(2-\sin\theta)$

$= mg(2-2+\sin\theta)$

$= mg\sin\theta$

$\cancel{R = \frac{1}{2}mg\sin\theta}$

$\sin\theta = \frac{2R}{mg}$

P remains in contact if $R \geq 0$

$\theta \leq \pi$

$\sin\theta \geq 0$

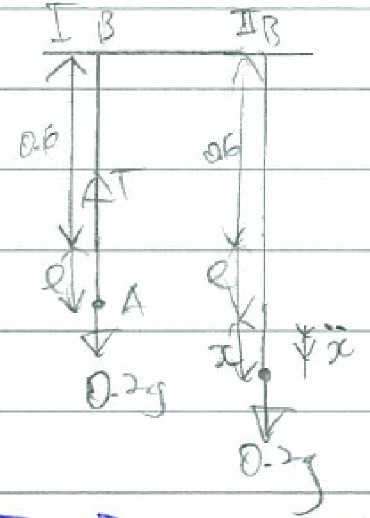
$\frac{2R}{mg} \geq 0$

$2R \geq 0$

$R \geq 0$

$\therefore$ Particle stays in contact

5.



a) $T = 0.2g$

$T = \dfrac{\lambda x}{a}$

$0.2g = \dfrac{48e}{0.6}$

$e = 0.0245$

$T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{20} = \dfrac{\pi}{10}$ s

$[F = ma]$

$0.2g - \dfrac{48(0.0245 + x)}{0.6} = 0.2\ddot{x}$

$0.2g - 1.96 - 80x = 0.2\ddot{x}$

$\ddot{x} = -400x$

∴ SHM with $\omega^2 = 400$

b) $v_{max} = \omega r =$ ~~...~~ $20 \times 0.2755 = 5.51 \text{ ms}^{-1}$

c) $x = 0.2755 \cos 20t$

$0.1255 = 0.2755 \cos 20t$

$\cos 20t = \dfrac{251}{551}$

$20t = 1.098, (2\pi - 1.098)$

$t_1 = 0.0549$

$\therefore t_2 = 0.259$

$T_{req} = 0.259 - 0.0549$

$= 0.204$ s

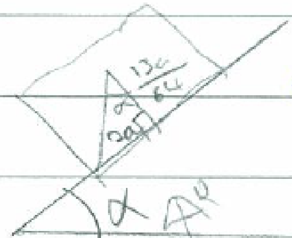
$\boxed{\text{NB Piston going eq clearer}}$

6. a) $\pi(2a)^2\cdot\frac{3}{2}a\cdot\frac{3}{4}a - \frac{2}{3}\pi a^3 \times \frac{3}{8}a = \left[\pi(2a)^2\cdot\frac{3}{2}a - \frac{2}{3}\pi a^3\right]\bar{d}$

$$\frac{36\pi a^4}{8} - \frac{6\pi a^4}{24} = \left(\frac{12\pi a^3}{2} - \frac{2\pi a^3}{3}\right)\bar{d}$$

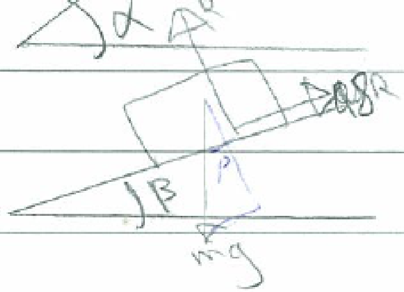
$$\frac{17\pi a}{4} = \frac{16\pi}{3}\bar{d}$$

$$\bar{d} = \frac{17\times 3a}{4\times 16} = \frac{51}{64}a$$

b) 

$$\tan\alpha = \frac{2a}{\frac{45a}{64}} = 2a\times\frac{64}{45a} = \frac{128}{45}$$

$$\alpha = 70.6^\circ \text{ (1dp)}$$

c) 

$R = mg\cos\beta$

$0.8R = mg\sin\beta$

$0.8\,mg\cos\beta = mg\sin\beta$

$0.8 = \tan\beta$

$\beta = 38.7^\circ \text{ (1dp)}$

7 a) $\frac{1}{2}mu^2 = \frac{1}{2}mv^2 - mg(a\sin\theta)$

$\frac{3}{2}ga = v^2 - 2ga\sin\theta$

$v^2 = \frac{3}{2}ga + 2ga\sin\theta$

b) $[F = ma]$

$T - mg\sin\theta = \frac{m}{a}\left(\frac{3}{2}ga + 2ga\sin\theta\right)$

$T - mg\sin\theta = \frac{3}{2}mg + 2mg\sin\theta$

$T = \frac{3}{2}mg + 3mg\sin\theta = 3mg\left(\frac{1}{2} + \sin\theta\right)$

c) When $\theta = 210$, $T = 3mg\left(\frac{1}{2} + \sin 210\right) = 3mg\left(\frac{1}{2} - \frac{1}{2}\right) = 0$

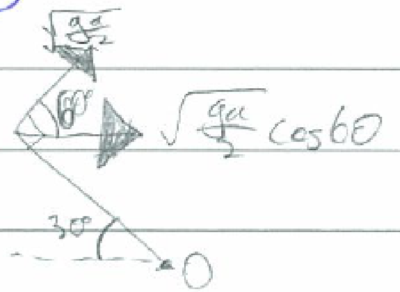
d) $\frac{1}{2}mu^2 = \frac{1}{2}\cdot\frac{3}{2}\cdot ga = \frac{3}{4}mga$ = KE initially

PE at the top: mga

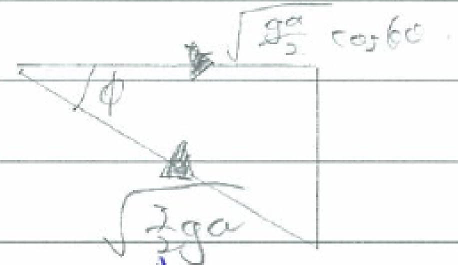
$\therefore$ P cannot complete a circle because it doesn't have through energy

e) No external work is done on the particle, so its mechanical energy is the same at the level of O

f) When $\theta = 210$, $v^2 = \frac{3}{2}ga + 2ga\sin 210 = (\frac{3}{2} - 1)ga = \frac{1}{2}ga$



At A:



$$\cos\phi = \frac{1}{2}\sqrt{\frac{ga}{2}} \times \sqrt{\frac{2}{3ga}} = \frac{1}{2\sqrt{3}}$$

$$\phi = 73.2° \text{ (1dp)}$$